**Partial differential equations**

**Homework**

**Task 7. Non-homogeneous string vibrating equation**

**Methodical instructions**

Consider the movement of the body under string exterior force characterized by the given function *f.* This is described by non-homogeneous vibrating string equation

*utt = a2 uxx* + *f*(*x*,*t*), 0 < *x* < *L*, *t* > 0.

Suppose the string is in the state of equilibrium and has zero velocity at the initial time. Then we have the initial conditions

*u*(*x*,0) = 0, *ut*(*x*,0) = 0, 0 < *x* < *L*.

The ends of the string are free, i.e. we have one of the following boundary conditions

*ux*(0,*t*) = 0, *ux*(*π*,*t*) = 0, *t* > 0.

**Table of parameters**

|  |  |  |  |
| --- | --- | --- | --- |
| variant | *L* | *a* | *f* |
| 1 | π | 3 | –cos *x* |
| 2 | 1 | 2 | –cos π*x* |
| 3 | 1 | 2 | cos 2π*x* |

**Actions**

It is necessary perform the following steps:

1. Determine the solution of the problem as cosine Fourier series.
2. Find the Fourier coefficient of the parameters of the system.
3. Solves ordinary differential equations with respect to the Fourier coefficients of the solution of the problem.
4. Check that this is, in reality, the solution of the boundary problem.
5. Show the graph (position of the string for the different time points).
6. Give the physical interpretation of the results.

Use the example from the lecture as the sample.